Sound and the Vibrating String

"Acoustics" is the branch of science that deals with sound. The science is very complex, but a little knowledge of the nature of sound, and in particular a vibrating string, can be very helpful in getting the most out of your guitar experience.

The “Waving” String

A string vibrates and creates a complex wave of energy pulses. These pulses are either transmitted to the ear through air or are translated as electrical pulses that are transmitted to a speaker system and then to the ear.

A string of fixed tension, density and length vibrates in a nearly continuous wave of sound as a function of amplitude, duration and frequency.

Continuous waves of sound have many properties and are studied in depth in the field of acoustics. A vibrating guitar string is not exactly a continuous wave but behaves close enough to be considered such. (Harvard Dictionary of Music)

“Amplitude” affects the degree of perceived loudness and is affected by the amount of energy used to set the string in motion.

“Duration” is the measure of length of time of vibration and is also affected by the amount of energy used to set the string in motion.

“Frequency” is measured in cycles per second called “hertz” and is not generally affected by the energy used to strike the string. The frequency of tones in music are referred to in terms of “pitch”.

The “fundamental frequency” is the frequency of the largest vibration. It is also the "perceived" pitch.

The audible frequency of sound for average humans is approximately 20 to 20,000 hertz.
Size, Length and Tension

“Tension” is the tightness of the string and affects frequency proportionally (the tighter the string - the higher the pitch.)

“Density” is the relative thickness of the string and affects frequency inversely (the bigger the string - the lower the pitch.)

“Length” is the distance from nut (or selected fret) to bridge and affects frequency inversely (the shorter the string - the higher the pitch.)

Harmonic Series

In addition to the wave of fundamental frequency, the string is also vibrating in a “harmonic series” of waves of integer multiples of the fundamental. “Overtones” refer to all the “harmonics” except the fundamental. The fundamental is referred to as the first harmonic. The second harmonic is therefore the first overtone.

1F

2F

3F

The fundamental and first two overtones of the Harmonic Series.

In mathematics, the "Fourier Theorem" states that any complex wave can be defined as the summation of a series of simple sine waves each with its own frequency, amplitude and duration. Fourier Analysis is used to study the acoustic properties of a musical instrument.

The complex wave formed by the combination of all the waves, each with its own amplitude, frequency and duration, produces the characteristic sound of the musical instrument.
**Timbre**

The qualities of the sound of a musical instrument resulting from the complex wave of relative amplitudes of the harmonic frequencies is referred to as “timbre” (pronounced tam-ber).

A vibration producing higher amplitudes of the higher frequency harmonics sounds more shrill (bright) and is referred to in music as having increased “treble”. On the guitar the higher overtones are louder near the bridge. To capitalize on this, an electric guitar may have "treble" pickups located nearer the bridge.

A "treble" knob on an electric guitar is designed to accentuate the higher frequencies.

A vibration producing higher amplitudes of the lower frequency harmonics sounds more mellow (dull) and is referred to in music as having increased "bass". The electric guitar can have a "rhythm" or "bass" pickup nearer to the neck and/or a "bass" control knob.

Timbre is affected by many things. As already stated, the string produces more higher overtones near the bridge (treble pickup) and more lower overtones near the neck (rhythm pickup).

Timbre can be affected acoustically or electronically.

The characteristics of the reflective medium of an acoustic guitar affect timbre. Larger body guitars generally have deeper tone than small bodies because they have increased resonance of the lower harmonic overtones. The type of wood and variations of top, back and sides can affect the tone. The bracing of the top affects tone. Variations included “scalloping” (adjusting the shape by carving) and placement configurations. C.F. Martin brought the now high-end standard "cross bracing" configuration to the guitar from his experience with making violins. The finish on the wood can affect the tone as well as the glue used to connect the pieces of the guitar. Many experts have studied for many years the composition of a Stradivarius violin trying to figure out what is the combination of characteristics that gives those instruments such a nice tone.

The timbre of an electric guitar is also affected by its body although in general not as much as that of an acoustic. The main two variations are “solid bodies” and “hollow bodies”. All else equal, the hollow bodies are generally more mellow.

Timbre is also affected by string composition and density. The newness of the string affects the timbre. Strings tend to go dull with time. One reason is that deposits can accumulate on the string and this affects density. It is therefore a good idea to wipe the strings down after playing them to prolong their life. Iron deposits (rust) caused by
finger perspiration will surely dull a string and shorten its useful life.

The timbre of a guitar can be affected in a large way by electronic filtering. On the guitar, the bass and treble tone adjustments, depending on design, either enhance or filter frequency. At the next level "equalizers" are processors either external or built into the guitar or amp which are designed to vary tone by multiple separate segments of frequency. They come in varying number of "bands" each with its own "bandwidth". The most common numbers of bands are three, eight, sixteen and thirty-two. “Parametric” equalizers provide another multi level of tone adjustment. Parametric EQ's have a control for bandwidth and usually have three or more tone centers from which to vary. Another type of processor to affect tone is the "exciter". The typical exciter’s marketing verbiage claims to "unmuddy" the sound. The overall affect however seems to be an adjustment to the high end of timbre.

Another way to electronically affect timbre is with variable tone adjustment processors. This includes "wah-wah" pedals, "flangers", "phasers" and the like. The wahs vary tone with a foot control. There are units that can simulate a wah sound but the variation is not "real-time" controllable like the pedal. The flangers vary tone and frequency by controlled adjustments involving pitch and delay. “Chorus” units vary frequency producing a swirl of changing timbre.

The timbre of a string vibration is affected by the medium providing the energy to set it in motion. A thicker pick will have a more mellow tone than a thin one. The composition of the pick such as a nylon, tortoise shell, plastic, metal, or fingernail and flesh affect the tone of the sound produced.

The location along the length of the string where the vibration is set in motion affects the timbre. The volume of the higher harmonics are increased near the string ends. The volume of the lower harmonics are increased as the striking point nears the middle of the string. Rhythm technique is generally enhanced by strumming the strings more in a position near the neck. Lead can be varied by changing the location of the strike of the string. The picking point increases treble near the bridge and mellows out near the neck. A very nice effect can be attained by pulling a pick in a reverse motion across the strings near the bridge. This produces a chime sound that is used on many ballads as an embellishment.
DERIVATION OF MUSICAL TONES

"THE SAME ONLY HIGHER"

As stated earlier, frequency is inversely proportional to string length. Among other things, this means that if a string is vibrating at a certain frequency and its length is shortened by one half, the new frequency of vibration will be twice that of the original. If we call the first frequency "F" for fundamental, the second frequency will be "2F" derived from the inverse proportionality of the length being one half.

Also as stated before, the string is vibrating in waves of the harmonic series and by lightly touching the string at the "node" points of the harmonics, several can be easily heard. In fact what can be demonstrated is that in doing so, you are eliminating at each node the lower nodes of vibration and what remains is all the rest. The relative volumes of the harmonics decrease and so what you perceive to hear is the harmonic at that node.

Therefore, another way to produce a frequency of "2F" with a string is to strike the string in such a way as to hear the second harmonic. This is by definition the integer two multiple of the fundamental. The point at which the string is vibrating in two waves is the half way point of the string. You can check this out with your ruler. In fact, the acoustical property of the second harmonic creates a very precise way to measure the halfway point of a vibrating string.

Listen to the tone made by lightly touching the strings exactly above the metal of the twelfth fret. This tone is called the "second harmonic" or "first overtone". The string is vibrating in a "series" of these tones in addition to the main tone which is called the "fundamental". The overall effect of these tones is called "timbre" (pronounced tam-ber) and contributes greatly to the sound of the guitar as well as to musical harmony in general. Try finding more of these tones especially above the seventh, fourth and fifth frets. They should be found at distances of equal ratios of the length of the overall string, i.e., 1/2, 1/3, 1/4, 1/5, etc.

The two tones, one made by shortening the length of the string by half and the other made by exciting the second harmonic are therefore each "2F" and are the same pitch although they vary slightly in timbre.

If we consider the harmonic series of these two differently produced tones of frequency "2F", we can reason that they are the same except for possible slight variations in timbre in the higher nodes of the series. The harmonic node points of the two waves will be the same over the length that is the same. All of the harmonics of the tone made by decreasing the length of the string by half are in the harmonically produced tone. The only tone missing is the fundamental which was used to start the process.
The frequency "2F", referred to as the "octave" in the music of western civilization, is considered the "same as "F" only higher". I assert that it is because the harmonics of the two previously derived tones are the same. On the guitar, the "2F" node point is directly above the metal of the twelfth fret. In acoustical analysis as well as music theory, the fact that frequencies in multiples of two are considered the same, is extremely important.

**INTONATION**

On the guitar the properties of the tone produced at the twelfth fret compared with the tone produced by the second harmonic forms the basis for adjustment of intonation. Intonation is the "in tuneness" of the guitar across the spectrum of the fretboard. Adjusting intonation should be done by a professional.

The formula is simple, however, the second harmonic should be in tune with the tone produced at the twelfth fret. If you change to a different gauge strings you may find that the tension on the neck changes and the neck may shift its natural bow. Basically when this happens the distance changes between the first and twelfth fret and the overall string length must be adjusted to accommodate the change. Hopefully, if this happens you have an adjustable bridge.

**ADDING MORE TONES**

In any musical system, whether it be ours (Western Civilization), Chinese, Hindu, African, Ancient Greek, Arab or whatever, the "different" tones can be defined in terms of being between some frequency and two times that frequency. Any thing else can be divided or multiplied by two enough times to fall into that category. Again, a fundamental property of sound in nature is the perception that frequency multiples of two are the same.

The next important node point on the harmonic series is the third harmonic. This produces a frequency of "3F". The node points are located by dividing the string in three equal parts. If we shorten the string to the point corresponding to two thirds the original length, we would be at the node of the third harmonic. The pitch at this length is three halves the fundamental. This pitch is not considered the same but is nevertheless a dominant tone relative to the fundamental. This tone in its simplest form relative to the fundamental is "3/2F". This pitch has been used to create the music systems of most of the civilizations of the world.
Greek Culture and the Pythagorean

The most ancient music system related to us was the Greek system, generally credited to the work of Pythagorus and his school of disciples. Although I had heard of Pythagorus, the mathematician studying geometry in high school (The Pythagorean Theorem), I first learned of Pythagorus, "the musician", in college physics. I was struck with curiosity about how a Greek philosopher/mathematician could learn so much about the properties of a vibrating string, 450 years B.C. I have since considered him the granddaddy of all guitarists or at least affectionately told my guitar students such.

The Pythagorean scale is derived mostly on the interval of "3/2F". Pythagorus discovered an interesting phenomena. If you start with a fundamental frequency and add tones that are three halves the previous tone and adjust these using the division by two principle, you will end up “almost” at "2F" after twelve such intervals. The Greeks called this succession of tones the "Chromatic" scale. The "almost" (called the Pythagorean Comma) is a difference of approximately twenty-four hundredths of the interval of "2F". This "almost" is also the reason for so many variations of scale derivation.

Pythagorus also experimented with subsets of the twelve tones and used a set of four tones called the "tetradchord" to create the basic unit of the Greek seven tone "diatonic" scale. The Greeks named the scale for the two types of intervals (whole step and half step) used to produce the scale. The whole step was defined as the distance between the four and fifth tones of the scale. The whole step occurred five times in the scale and a lesser interval of half the frequency occurred twice. We still use the same name for our seven tone scale.

The interval of "2F" became the eighth note of the scale and was thus given the name of "octave". The interval of three halves above was the fifth tone of the diatonic scale and is now called the "fifth". The interval of two thirds below was the fourth tone of the scale and was called the “fourth”. The interval of "2/3F" below is the tone that has the fundamental as its fifth above since "2/3 * 3/2F" equals "F".

The school of Pythagorus also studied the properties of two strings of similar characteristics vibrating together. He notice that the most pleasing ratios were in small whole number ratios such as 2:1, 3:2, 4:3, 5:4, ... 9:8, etc. Combining tones according to the harmonic series and these ratios produced pleasing complex tones called "chords" and were said to be in "harmony".

The "circle" produced by the interval of "3/2F" is thus called the "Cycle of Fifths". Chords seem to progress most harmoniously along this circle.
Chinese Culture and the Pentatonic

Pythagorus was not the first “guitarist” relative to all musical cultures, however, since he was probably influenced by the Arabs. But way before that, the Chinese, as many as 5000 years ago or more were also doing experiments with strings. They derived a scale of five tones (pentatonic) also using the interval of three halves. They chose five tones apparently because of their culture's importance of the five substances of earth, wind, fire, water and air. The interesting thing about the Chinese was that their starting note (the fundamental) was so important to their culture. Each "dynasty" had its own way of “spiritually” coming up with the beginning tone. In one case it amounted to the tone produced by striking a cylinder of a particular length filled with a particular amount of grain.

Bach and Equal Temperament (A balanced compromise)

The chromatic twelve tone scale commonly used today evolved from the Greeks and Arabs into what is now generally considered the music of "Western Civilization". It has been derived generally from a system of notes based on the three halves intervals but "adjusted" so that the above "Pythagorean Comma" is distributed evenly across all the tones. Each "equal tempered" three halves is therefore approximately two one hundredths (cents) "out of tune" with the natural three halves system of the harmonic series. This balanced “out of tune but close enough” scale was the product of several permutations but has been credited to Johann Sebastian Bach for its acceptance when he published "The Well Tempered Clavier" which was a systematic set of musical pieces each in the key of the twelve different tones.

Just and Mean Tone

Two other scale derivations are worthy of consideration in analyzing our current musical structure. These scales include "Just Tone" scale and the "Mean Tone" scale. The just tone scale is designed to include notes more in tune with the harmonic series. The mean tone scale is an attempt to distribute the error of the "Pythagorean Comma" in a way that still has perfectly in tune fifths and fourths.

To further look at these scales we must consider a few more tones in the harmonic series. So far, we have discussed "2F" and "3F". "4F" is again a multiple of two and is the same as the fundamental only higher. "5F" is the point where the next most interesting harmonic occurs. If you use the multiple of two principle the tone can be reduced between "F" and "2F" to "5/4F". The tone of "5/4F" is considered the third tone of the diatonic scale. This again was one of the "pleasing ratios" of Pythagorus.
If you take the three different tones produced by "F", "3/2F" and "5/4F", which are included as the first three different tones of the harmonic series and play them together you will have what is considered now as a "Just Tuned" major chord. If you take the same three tones of the tone produced at "3/2F" and the three at "2/3F", you will have seven different tones which will produce the "Just Tone" diatonic scale.

If you derive the same scale using the cycle of fifths, the thirds will be out of tune with the same tone used as a fifth. This is the reason for the "mean tone" scale. The mean tone scale divides the distance of the third above and the third below equally and thus is an average or mean.

The equal tempered scale probably won out because at the critical time, instrument manufacturing could not easily accommodate a system that had to accommodate for so much out of tuneness. Today however, there is a resurgence of "Just Tone" tuning since the digitally produced sounds of computers and synthesizers can pretty much accommodate any exception. There already have been great "Just Tone" composers and I'm sure more to follow. There is at least one Web Site on the Internet that is dedicated to "Just Tuning".

"Norjust Tuning"

While considering the various derivations for harmonic tunings, I experimented and came upon the fact that the first three unique tones of the harmonic series of a fundamental and a fifth, produce a major ninth chord. To understand this, consider a guitar with a sixth string tuned to a relative "D" as our fundamental. I say relative because unlike the Chinese, it doesn’t really matter what the fundamental frequency is in hertz, but close enough to standard pitch so that the string and guitar produce a pure tone. The “fifth” (3/2F) relative to “D” is a just “A”.

As I have discussed, when these strings vibrate, they do so in a wave of frequencies corresponding to the harmonic series. Based on the timbre produced by the instrument, the relative volumes of the overtones will vary but can be heard by slightly touching the string at the node points.

As the sixth string vibrates, it produces a fundamental which is the wave produced by the length of the whole string moving back and forth. The first overtone (second harmonic) is the wave of the string vibrating in two halves. Its frequency is 2F. This pitch is considered the same as the fundamental, only higher.

The next overtone is the third harmonic produced by the wave of the string divided in thirds. This pitch is the first overtone that is considered different. Its frequency is 3F. We know that frequencies of multiples of two are considered the same and therefore find that the unique frequency closest to our fundamental is 3/2F. For discussion relative to our standard music naming conventions, this note is referred to as the “fifth”
and in relative pitch to our fundamental “D”, it is a “just A”. This pitch is an almost imperceptible two cents different than the equal tempered “A” relative to our “D”.

Doing a similar procedure for 4F, we find that divided by two our fourth harmonic is just another two multiple of our fundamental. Our next unique tone comes with the fifth harmonic. The string divided in fifths vibrates five times faster than our fundamental. Again, by reducing the tuples, we find the unique frequency of 5/4F. This pitch relative to “D” is “F#“. As a side note, this frequency is noticeably different than the equal tempered “third” as well as greatly different from the Pythagorean “third” (3/2*3/2*3/2*3/2*3/2=81/64).

Doing the same procedure with the fifth string, the three tones derived will be “A”, “E”, and “C#”. All six tones together (“D” and just tone’s “F#”, “A”, “C#” and “E”) combine to form a just major ninth chord.

I decided to pursue this and tuned my guitar so the sixth string was tuned to “relative D”. The fifth string was tuned by the third harmonic of "D" which was “just A” and not the equal tempered “A” which is off by two cents. The fourth string was tuned to the third harmonic (just fifth) of the fifth string producing a just “E”. The third string was tuned “justly” at the second fret to the octave (second harmonic) of the fifth string. The second string second fret was tuned to the “just third” (fifth harmonic of the fifth string which is near the fourth fret) producing a relative “C#” note. The first string second fret was tuned to the “just third” of the fourth string and thus was a just “F#”.

When played while fretting the first, second, third and fourth strings on the second fret a lush "D major ninth" chord is produced. Since the intonation is still standard, to do more you have to be careful of which other notes to play together. You can also play the lush major ninth chord at any other fret using barred first and third fingers. If you think about it, this barred chords will be in just tune relative to each string but not relative to each other since the frets are positioned by equal temperament. This is not noticeable, and especially the I – IV – V chords will be too close to complain.

This tuning is not very useful for much more than an experiment in just tone tuning but is possibly worth the effort to try it so you can experience the difference between out of tune thirds and justly tuned thirds. Another variation of Norjust calls for tuning the fourth string to the octave of "D". A chord with a three string bar of the second fret and the three open lower strings then produces the "major seventh" of the fundamental and fifth of the harmonic series.